Friday, September 4, 2015

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Problem 2

Problem. Show that $f(x) = 3 - 4x$ and $g(x) = \dfrac{3-x}{4}$ are inverse functions.

Solution. The domain and range of f are both the set of all real numbers. The domain and range of g are also both the set of all real numbers. Furthermore,

$$\begin{aligned} f(g(x)) &= 3 - 4g(x) \\ &= 3 - 4\left(\frac{3-x}{4}\right) \\ &= 3 - (3 - x) \\ &= x \end{aligned}$$

and

$$\begin{aligned} g(f(x)) &= \frac{3 - f(x)}{4} \\ &= \frac{3 - (3 - 4x)}{4} \\ &= \frac{4x}{4} \\ &= x. \end{aligned}$$

Problem 5

Problem. Show that $f(x) = \sqrt{x-4}$ and $g(x) = x^2 + 4$, $x \geq 0$ are inverse functions.

Solution. The domain of f is $\{x \mid x \geq 4\}$, which is the range of g, and the domain of g is $\{x \mid x \geq 0\}$, which is the range of f ($f(x)$ is the *nonnegative* square root). Furthermore,

$$\begin{aligned} f(g(x)) &= \sqrt{g(x) - 4} \\ &= \sqrt{(x^2 + 4) - 4} \\ &= \sqrt{x^2} \\ &= x. \text{ (because } x \text{ is nonnegative)} \end{aligned}$$

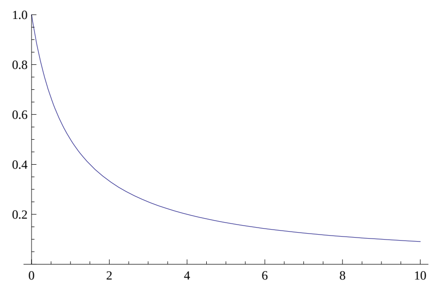
and

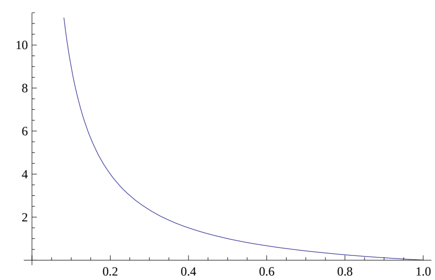
$$\begin{aligned} g(f(x)) &= (f(x))^2 + 4 \\ &= (\sqrt{x-4})^2 + 4 \\ &= (x-4) + 4 \\ &= x. \end{aligned}$$

Problem 8

Problem. Show that $f(x) = \dfrac{1}{1+x}$, $x \geq 0$, and $g(x) = \dfrac{1-x}{x}$, $0 < x \leq 1$ are inverse functions.

Solution. To verify the domains and ranges, it might be helpful to draw their graphs.





We see from the graphs that the domain of f is the range of g and the domain of g is the range of f.

Furthermore,

$$\begin{aligned} f(g(x)) &= \frac{1}{1+g(x)} \\ &= \frac{1}{1+\left(\frac{1-x}{x}\right)} \\ &= \frac{x}{x+(1-x)} \\ &= \frac{x}{1} \\ &= x. \end{aligned}$$

and

$$\begin{aligned}
g(f(x)) &= \frac{1 - f(x)}{f(x)} \\
&= \frac{1 - \left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)} \\
&= \frac{(1 + x) - 1}{1} \\
&= \frac{x}{1} \\
&= x.
\end{aligned}$$

Problem 10

Problem. Match the graph of the function with the graph of its inverse function.

Solution. This graph is matched with graph (b). This graph passes through the points $(0, 2)$ and $(6, 0)$ and graph (b) passes through points $(2, 0)$ and $(0, 6)$.

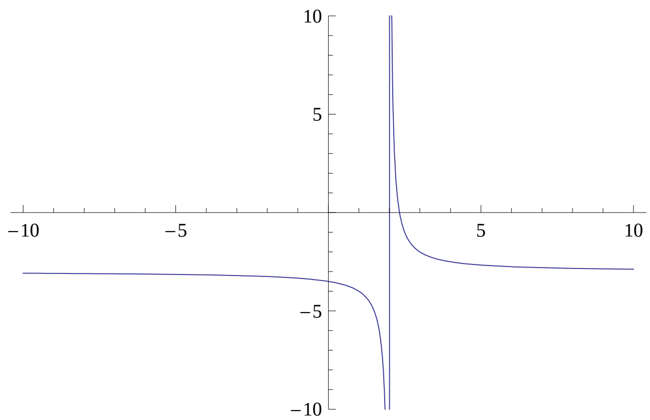
Problem 17

Problem. Use the Horizontal Line Test to determine whether the function

$$h(s) = \frac{1}{s - 2} - 3$$

is one-to-one on its entire domain.

Solution. The domain of $h(s)$ is $\{s \mid s \neq 2\}$. The graph is



From the graph we can see that it is one-to-one. (The vertical blue line is an artifact of the software.)

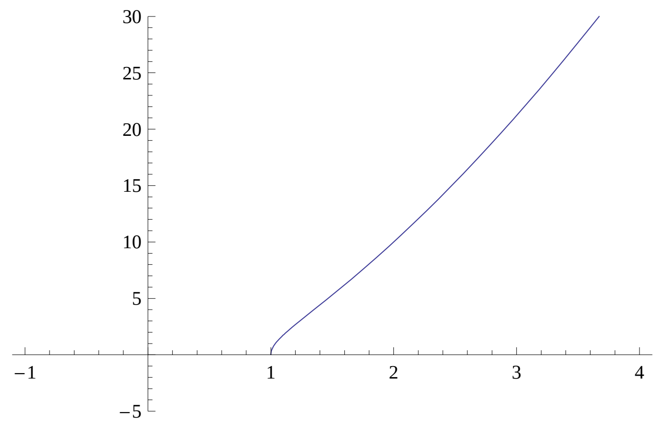
Problem 20

Problem. Use the Horizontal Line Test to determine whether the function

$$f(x) = 5x\sqrt{x-1}$$

is one-to-one on its entire domain.

Solution. The domain of $f(x)$ is $\{x \mid x \geq 1\}$. The graph is



From the graph we can see that it is one-to-one on its domain.

Problem 29

Problem. Show that $f(x) = (x-4)^2$ is strictly monotonic on the interval $[4, \infty]$.

Solution. We find that $f'(x) = 2(x-4)$ and it is easy to check that $2(x-4) \geq 0$ when $x \geq 4$. Therefore, $f(x)$ is monotonic increasing on $[4, \infty]$.

Problem 33

Problem. Show that $f(x) = \cos x$ is strictly monotonic on the interval $[0, \pi]$.

Solution. We find that $f'(x) = -\sin x$. We know that $\sin x \geq 0$ when $0 \leq x \leq \pi]$, so $-\sin x \leq 0$ on that interval. Therefore, $f(x)$ is monotonic decreasing on $[0, \pi]$.

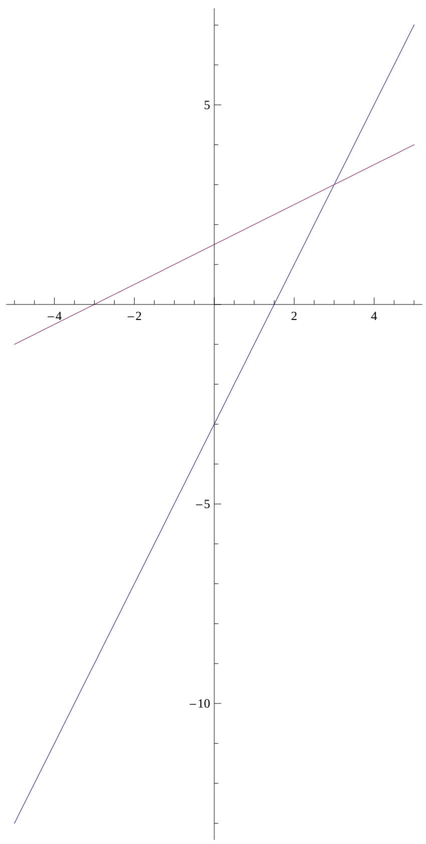
Problem 35

Problem. Find the inverse function of $f(x) = 2x - 3$, graph f and f^{-1}, and state the domain and range of f and f^{-1}.

Solution. Write $y = 2x - 3$, swap x and y, and solve for y.

$$\begin{aligned} x &= 2y - 3 \\ 2y &= x + 3 \\ y &= \frac{x+3}{2}. \end{aligned}$$

So $f^{-1}(x) = \dfrac{x+3}{2}$. The graphs are



The domain and range of both f and f^{-1} are the set of all real numbers.

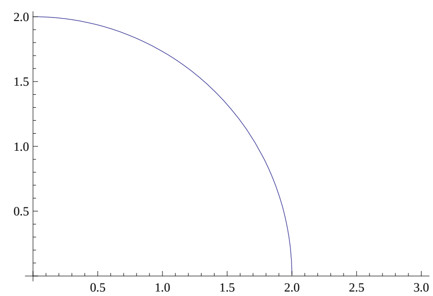
Problem 41

Problem. Find the inverse function of $f(x) = \sqrt{4 - x^2}$, $0 \leq x \leq 2$, graph f and f^{-1}, and state the domain and range of f and f^{-1}.

Solution. Write $y = \sqrt{4 - x^2}$, swap x and y, and solve for y.

$$\begin{aligned} x &= \sqrt{4 - y^2} \\ x^2 &= 4 - y^2 \\ y^2 &= 4 - x^2 \\ y &= \sqrt{4 - x^2}. \end{aligned}$$

So $f^{-1}(x) = \sqrt{4 - x^2}$. This is the same as f, so the two graphs are the same. The graph is



The domain and range of both f and f^{-1} are the set $\{x \mid 0 \leq x \leq 2\}$.

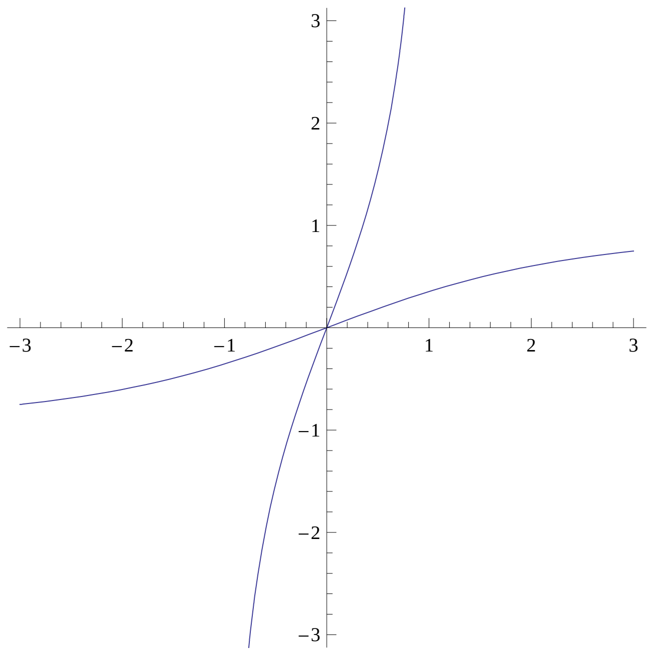
Problem 45

Problem. Find the inverse function of $f(x) = \dfrac{x}{\sqrt{x^2 + 7}}$, graph f and f^{-1}, and state the domain and range of f and f^{-1}.

Solution. Write $y = \frac{x}{\sqrt{x^2+7}}$, swap x and y, and solve for y.

$$\begin{aligned} x &= \frac{y}{\sqrt{y^2 + 7}} \\ x^2 &= \frac{y^2}{y^2 + 7} \\ x^2(y^2 + 7) &= y^2 \\ x^2y^2 + 7x^2 &= y^2 \\ x^2y^2 - y^2 &= -7x^2 \\ y^2(x^2 - 1) &= -7x^2 \\ y^2 &= \frac{7x^2}{1 - x^2} \\ y &= \frac{\sqrt{7}x}{\sqrt{1 - x^2}}. \end{aligned}$$

So $f^{-1}(x) = \dfrac{\sqrt{7}x}{\sqrt{1-x^2}}$. The graphs are



The domain and range of f are all real numbers and the interval $(0, 1)$, respectively. The domain and range of f^{-1} are the interval $(0, 1)$ and all real numbers, respectively.